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RESEARCH ARTICLE

# Acoustic Whispering Gallery Modes in a Split Ring Resonator

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**ABSTRACT:** In this work the Finite Element Method was used to numerically investigate the resonant frequencies for the acoustic Whispering Gallery Modes in a split ring type resonator. This is the case where a defect is considered in the ring resonator and the study was focused on how the resonant frequencies are affected by the presence of the defect in the ring. The findings of this study show that a kind of degeneration appears on the modes of the ring due to the insertion of the defect. The Acoustic Quality factors of the resonances were also calculated and were found to have high values proving a strong enhancement of the field. The strong enhancement of the field is also followed by the observation of intense localization of it into the defect. The results of our study show strong evidence that the proposed structure could be a very promising candidate for applications based on acoustic signals such as sensors and filters.

**Keywords:** Whispering Gallery Modes, Acoustic Resonators, Acoustic Quality Factor, Finite Element Method

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## 1. INTRODUCTION

The manipulation of elastic or acoustic waves is of fundamental importance in the research area of wave propagation in nature. Controlling elastic or acoustic waves is a challenge which could provide great opportunities evolving technological applications in almost all range of phononic spectrum [1]. Phononic crystals are artificially made composites which, due to their periodic arrange of inclusions in a host material, are able to provide frequency areas where wave propagation is totally forbidden [2]. This special property of phononic crystals makes those composites very promising candidates for applications such as waveguides, resonant cavities, sensors, filters and seismic shields [3]. More recently acoustic metamaterials were another achievement in the field which introduced new functionalities not found in previous efforts [4].

In the field of Acoustic Resonators one of the most interesting phenomena is the well-known Whispering Gallery Modes (WGM). Historically, WGM were first studied by Lord Rayleigh for the case of sound waves propagating close to the cylindrical wall in St Paul's Cathedral, London [5-7]. Even though the first studies of WGM were for sound waves the scientific interest the years after the studies of Lord Rayleigh was mostly attracted by the relevant phenomenon in optics. Thus, there are only a few works including pure acoustic WGM. Due to the large enhancement of the electromagnetic field inside the resonator, the achievements of very small mode volumes and the very narrow line widths, optical WGM resonators have been a very good candidate over years for plenty of applications (lasers, filters, optical interconnects, etc.). So there are plenty of scientific works evolving WGM resonators and lots of applications have been proposed based on those structures. An optical resonator based on the circulation of light in dielectric volumes enables storage of optical power and is the basic idea for a wide range of fields such as quantum electrodynamics [8], photonics [9-11], biosensing [12], non-linear optics [13] and filters [14]. In addition, studies of optical resonators in glass microspheres [15], microrings [16], and microtoroids [17] gave rise to the applications afforded by the extremely long lifetime WGM supported by these structures. Aravantinos-Zafiris and Sigalas investigated numerically a structure which could be considered as a combination of a slot waveguide into a disk resonator and could be used for optical interconnects applications [18].

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Recently acoustic WGM in optical resonators were excited due to stimulated Brillouin Scattering. Bahl *et al.* numerically calculated the forms and frequencies of mechanical whispering-gallery modes in silica shells [19]. Sturman and Breunig investigated analytically and numerically acoustic WGM in spherical and cylindrical resonators made of an isotropic solid-state material without singling out pure longitudinal modes [20]. Kaproulias and Sigalas studied numerically with the Finite Difference Time Domain (FDTD) method the resonant modes of elastic waves in disk resonators [21]. Li *et al.* studied WGM in a micro-quartz tube immersed in liquid medium, excited by the evanescent wave of a Lamb wave device [22]. Jin *et al.* introduced the existence of WGM of a phononic crystal plate with hollow pillars [23]. Rostami-Dogolsara *et al.* studied an acoustic add-drop filter composed of two line-defect waveguides coupled through a ring resonator cavity all based on a phononic crystal platform [24]. Recently, Muhammad *et al*. studied the propagation of surface waves on semi-infinite silicon substrate with hollow silicon pillars under the existence of WGM in the pillars [25].

In this work by using the Finite Element Method (FEM), included in COMSOL Multiphysics® software package, the acoustic WGM in a ring resonator were studied. It was also examined the case where a defect is considered in the ring resonator and how the resonances are affected by the presence of the ring. This type of defected ring is similar to the split rings which were considered in electromagnetic metamaterials applications [26]. The Acoustic Quality factors ($Q_a$) of the resonances were calculated and found to have high values and, most importantly, highly localized displacements in materials with low sound velocities. The Acoustic Quality factor is defined as the ratio f/Δf where f is the central frequency of the resonance and Δf is the full width at half maximum of the transmission peak. The results of this work show that due to their high $Q_a$ values this kind of resonators could be a very promising candidate for several kinds of applications based on WGM devices.

## 2. RESULTS AND DISCUSSION

The studied structure consists of a Platinum ring resonator which is laying at the xy plane with outer radius $R_{out}$=0.1m and inner radius $R_{in}$=0.08m. The thickness of the ring considered along z axis is h=0.025m. The considered defect has width which is symbolized as w. Figure 1 shows all the geometric parameters of the studied structure. The ring is placed into the air and Perfectly Matched Layers are considered at the boundaries of each side of the simulation domain. It is also important to mention that for all calculations of this study at least 54995 elements were used.

Figure 2(a) shows the 5$^{th}$ order WGM resonance in this ring resonator found at frequency f=18072 Hz. The calculated Acoustic Quality factor for this case is $Q_a$ =18072. The 6$^{th}$ order WGM resonance for the ring, shown in Figure 2(b), was found at frequency f=20590 Hz and the calculated Acoustic Quality factor for this case is $Q_a$ =41180. It is important to be mentioned here that the plots indicate the total displacement. But from the analysis came up that only z displacement is taking place since x and y displacements are almost zero.

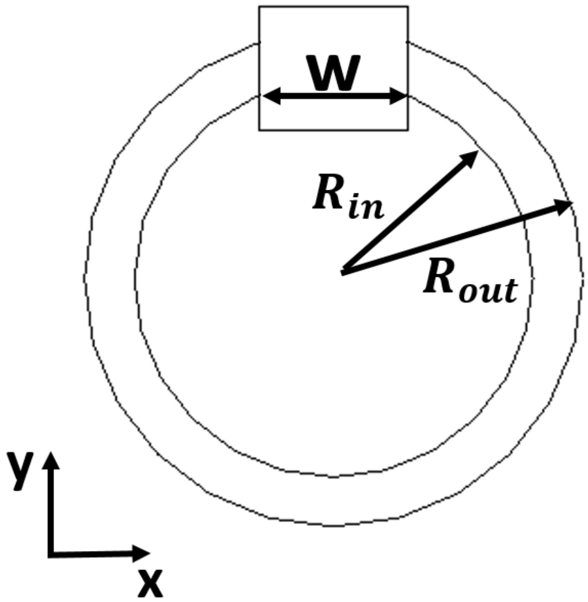


**Fig. 1.** The geometric parameters of the ring resonator structure laying on xy plane.

Another interesting case is the one where a defect was introduced in the ring. The defect which was placed in the ring was an air slot thus resembling the so called split ring resonator. The width of the slot, as already mentioned, is symbolized as w. Assuming width of the slot equal 0.06 m, for the 5$^{th}$ order resonance was found that that $Q_a$ =582 at frequency *f*=19777 Hz, as shown in Figure 3(a). What is also noticeable for this case is that the maximum displacement seems to be concentrated into the air slot. On the other hand for the same width of the defect there is another 5$^{th}$ order resonance at frequency *f*=18684 Hz where $Q_a$ = 8493. For this case of WGM resonance the displacement isn't into the air slot but mostly concentrated at the two sides of it as shown in Figure 3(b).

When reducing the width of the defect to w=0.02 m the 5th order WGM resonance appears at *f*=18594 Hz. Figure 4(a) shows this case of the defected ring resonator. The Acoustic Quality factor for this case of resonance was calculated $Q_a$ =2324 and the displacement field, mostly at z direction again, was found into the air slot. On the other hand there is another 5$^{th}$ order WGM resonance shown in Figure 4(b) for frequency *f*=18062 Hz where the maximum displacement is localized mostly into the ring and $Q_a$ =10625. Further reducing the width of the defect to w=0.01 m there are again two resonance frequencies for the 5$^{th}$ order WGM.

 

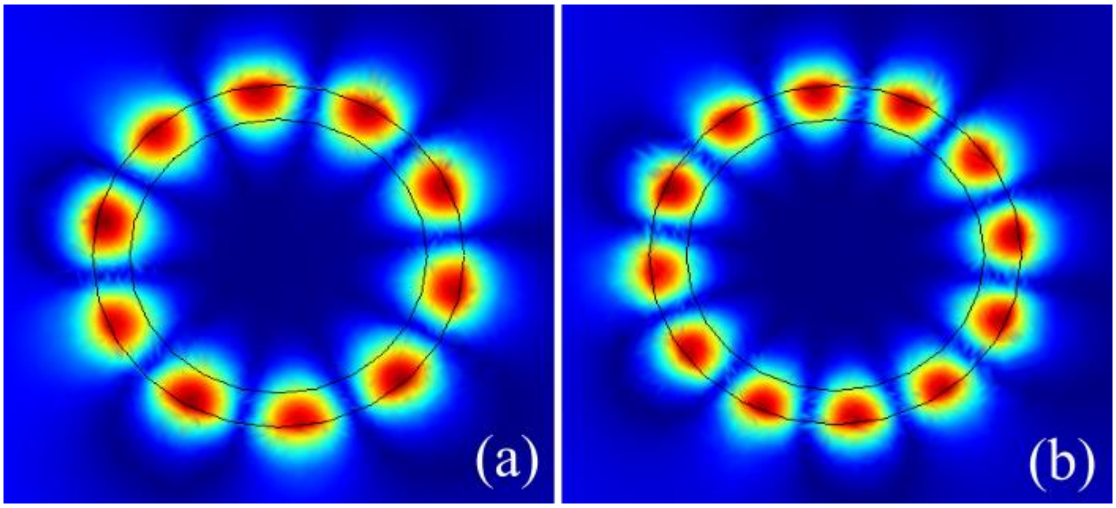


**Fig. 2.** The 5th order (a) and the 6th order (b) WGM of the non-defected ring resonator.

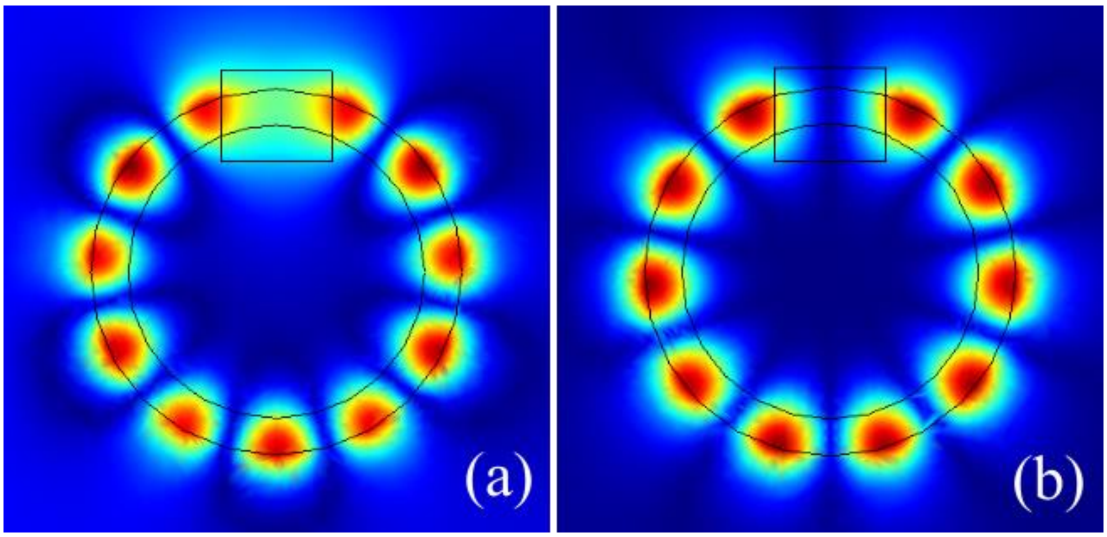


**Fig. 3.** The 5th order WGM resonance of the ring resonator when a defect of width w=0.06m is considered. (a) The resonance frequency is *f*=19777 Hz and (b) the resonance frequency is *f*=18684 Hz.

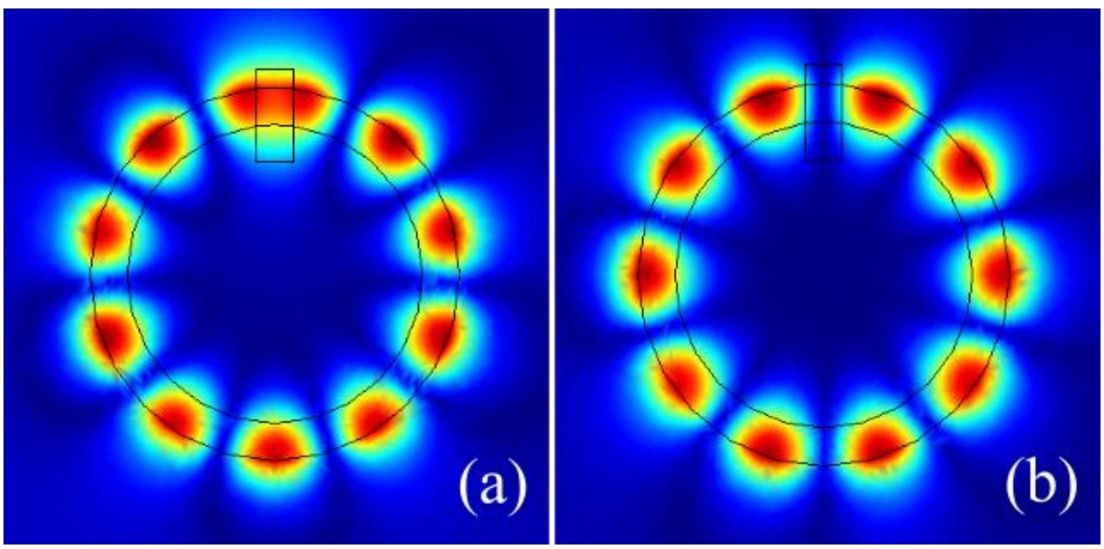


**Fig. 4.** The 5th order WGM resonance of the ring resonator when a defect of width w=0.02 m is considered. (a) The resonance frequency is *f*=18594Hz and (b) the resonance frequency is *f*=18062Hz.

 

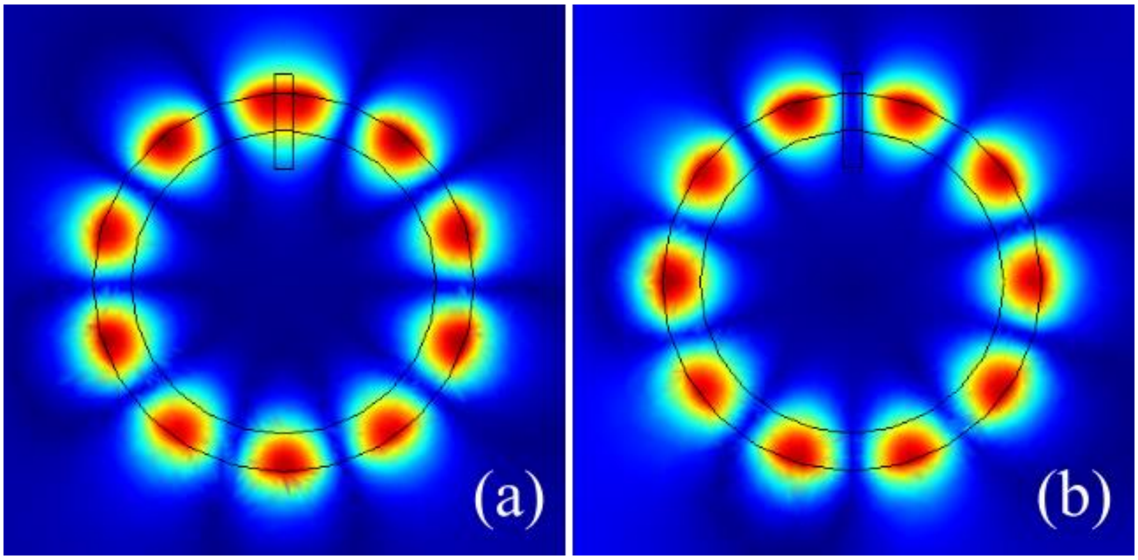


**Fig. 5.** The 5th order WGM resonance of the ring resonator when a defect of width w=0.01 m is considered. (a) The resonance frequency is *f*=18237 Hz and (b) the resonance frequency is *f*=17958 Hz.

Figure 5 shows the cases where width of the defect is w=0.01 m. For frequency *f*=18237 Hz there is a 5th order WGM resonance (Figure 5a). At this frequency the maximum displacement is localized into the air slot and has mostly z polarization. It is also important to be mentioned that the quality factor for this resonance was calculated $Q_a$ =16580. On the other hand there is another resonance at frequency *f*=17958 Hz where $Q_a$ = 9977. For this case of WGM resonance the field is mostly concentrated at the two sides outside the air defect and has orientation along z axis as shown in Figure 5(b). So, effectively, the 5th order resonance of the unperturbed ring splits into two resonances in the split-ring case. The low frequency resonance, where the displacement is mostly concentrated in the ring and has higher $Q_a$ value; and a higher frequency resonance with a high displacement inside the air slot and lower $Q_a$ value. As the width of the slot decreases, the frequency difference of the two resonances decreases.

Another interesting case that was numerically examined is the one were a glass substrate was considered below the defected ring resonator. The defect in the ring is considered again as an air slot of width 0.01 m. The substrate was a rectangular parallelepiped plate which had length 0.24 m, height 0.15 m and its width was 0.01 m. The plate was placed vertical to the plane of the ring resonator so that there are two points at the ring of width 0.01 m that the substrate plate and the ring resonator are touching. The 5th order WGM resonance was found at frequency f=18250 Hz and the Acoustic Quality factor was calculated $Q_a$ = 2281. For this case of the WGM resonance the field is concentrated into the defect slot as shown in Figure 6. It is also obvious that there is a decrease in the value of the quality factor due to the leakage of the field into the glass substrate. Similar to the case were a glass substrate was considered below the ring is the one were a small platinum (Pt) disk was considered at the base of the ring resonator.

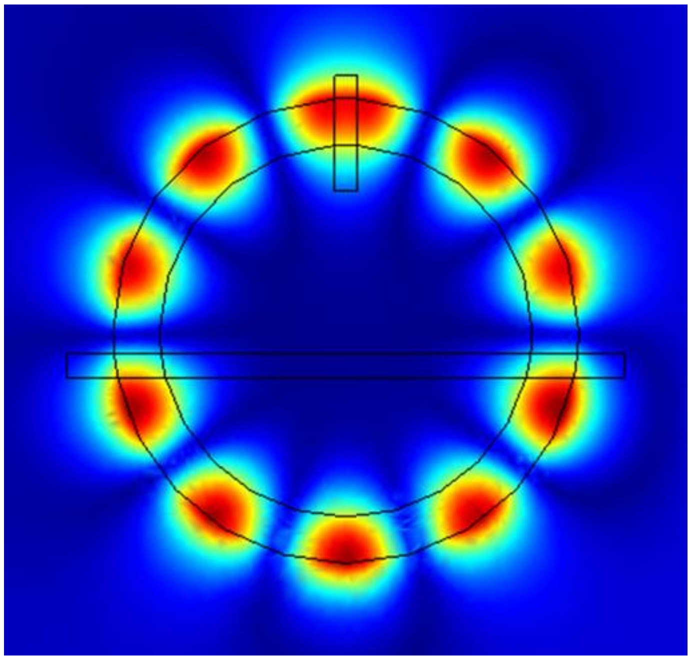

**Fig. 6.** The 5th order WGM resonance of the ring resonator when a defect of width w=0.01 m and a glass substrate were considered. The resonance frequency is *f*=18250 Hz.

The disk is made from the same material as the ring and has a radius equal to the inner radius of the ring. So the Platinum disk has radius R=0.08 m and its thickness is h=0.005 m. Figure 7 shows this modification of the structure. The insertion of the platinum disk actually creates a kind of toroidal ring resonator and the resonances of the defected ring were calculated when this structure is considered [17].

 

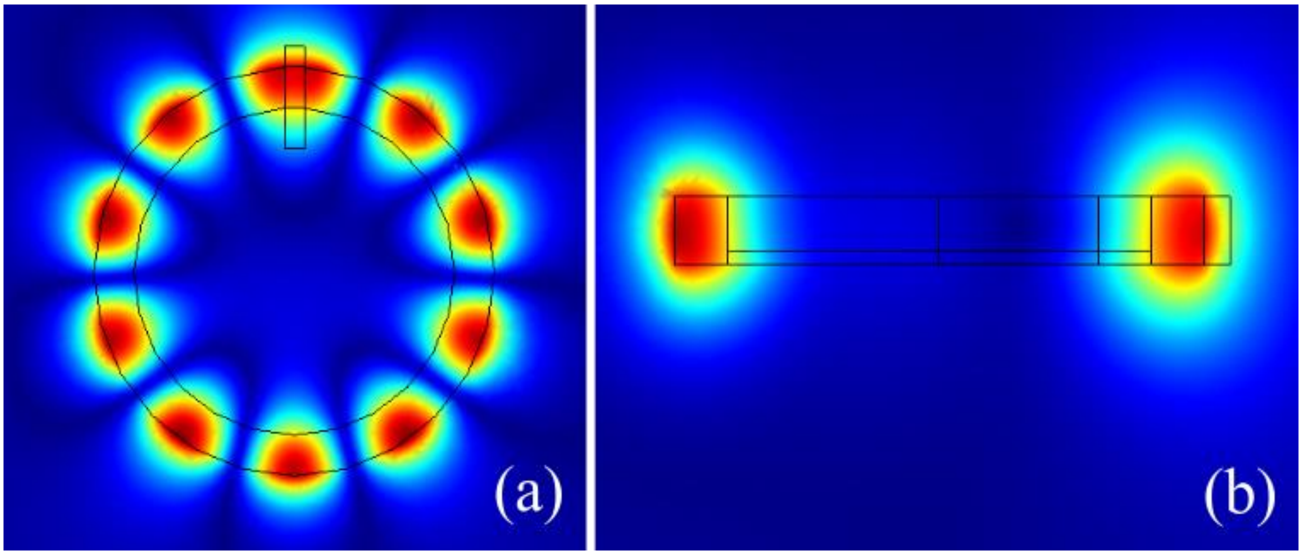


**Fig. 7.** The 5$^{th}$ order WGM resonance of the ring resonator when an air defect of width w=0.01 m and a Pt disk at the base of the ring. The resonance frequency is *f*=17714 Hz. (a) The xy plane of the structure and (b) the yz plane of the structure where the Pt disk at the base of the ring is indicated with the grey line.

**Table 1**. Collected results of the split ring acoustic resonator study.

| Defect width w (m) | Order of WGM | Frequency Resonance (Hz) | Acoustic Quality factor ($Q_a$) |
|---|---|---|---|
| **0** | 5$^{th}$, 6$^{th}$ | 18072 | 18072 |
| | | 20590 | 41180 |
| **0.06** | 5$^{th}$ | 19777 | 582 |
| | | 18684 | 8493 |
| **0.02** | 5$^{th}$ | 18594 | 2324 |
| | | 18062 | 10625 |
| **0.01** | 5$^{th}$ | 18237 | 16580 |
| | | 17958 | 9977 |
| **0.01 (with glass substrate)** | 5$^{th}$ | 18250 | 2281 |
| **0.01 (with Pt disk substrate)** | 5$^{th}$ | 17715 | 7381 |

For this structure the calculations resulted in a 5$^{th}$ order WGM resonance at frequency f=17715 Hz. The Acoustic Quality factor for this resonance is $Q_a$ = 7381. Additionally for this set of calculations the field is concentrated into the air slot defect. The Acoustic Quality factor for the relevant case without the disk but with the same width of the air slot defect was calculated $Q_a$ = 16580.

Although there is a decrease to almost the half value of the quality factor it is still a high value indicating strong enhancement of the displacement into the air slot. As in the case of toroidal ring resonator [17], the present split ring with Pt-base can stand on a glass tip located at the center of the base where the displacements are negligible (Figure 7(a)) and the calculations show that the resonances are not affected. Table 1 contains the collective results of our study for all the examined cases, in order to have a better view of the behavior of the structure according to the several modifications of it.

## 4. CONCLUSIONS

Summarizing, in this work the well-known WGM of an acoustic split ring resonator were numerically studied. Both cases with and without a defect in the ring took place in the calculations. The defect was considered as a small slot of air in the ring. In order to have a quantitative measure of those resonances and how the displacements where allocated and enhanced into the air slot or into the ring the Acoustic Quality factor for each resonance frequency was also calculated. For the case where the ring has no defect the acoustic WGM appeared. For the frequency range where our study was focused the 5$^{th}$ and 6$^{th}$ order of WGM resonances appeared with relatively high values of their Acoustic Quality factors.

The insertion of an air slot defect in the ring causes a kind of degeneration on the WGM resonances. There are two frequency resonances where, for the lower one, the displacement is maximized into the air slot while for the higher one the displacement becomes maximum into the ring. As the width of the defect decreases the enhancement into the air slot becomes stronger leading to higher values for the Acoustic Quality factors. On the other hand the resonances for which the field is mostly localized into the ring are not seem to be affected by the width of the defect. In all examined cases the Acoustic Quality factor for the relevant case had about the same value. The consideration of a substrate in the studied structure leads, as expected, to lower values of the Acoustic Quality factors due to the leakage of the acoustic energy into the substrate. The case of a Pt disk at the base of the split ring resonator seems to be more interesting since the value remains high for the case where the displacement is maximum into the air slot. Our numerical results provide for the first time strong evidence that the proposed acoustic split ring resonator could be a very promising candidate for applications evolving acoustic signal processing such as sensors and filters. In addition, the higher frequency resonance has strong enhancement of the displacement in the air (or other fluids) and for that reason may find applications on sensors or filters.

## CONFLICT OF INTEREST

The authors declare that there is no conflict of interests.

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