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RESEARCH ARTICLE

Dissipative Dynamics of an Interacting Spin System with Collective Damping

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ABSTRACT: The competition between Hamiltonian and Lindblad dynamics in quantum systems give rise to non-equilibrium phenomena with no counterpart in conventional condensed matter physics. In this paper, we investigate this interplay of dynamics in infinite range Heisenberg model coupled to a non-Markovian bath and subjected to Lindblad dynamics due to spin flipping at a given site. The spin model is bosonized via Holestien-Primakoff transformations and is shown to be valid for narrow range of parameters in the thermodynamic limit. Using Schwinger-Keldysh technique, we derive mean field solution of the model and observe that the system breaks z_2- symmetry at the transition point. We calculate effective temperature that has linear dependence on the effective system-bath coupling, and is independent of the dissipation rate and cutoff frequency of the bath spectral density. Furthermore, we study the fluctuations over mean field and show that the dissipative spectrum is modified by $O\left(\frac{1}{N}\right)$correction term which results change in various physically measurable quantities.

Keywords: Dissipative Dynamics, Spin Model, Quantum System, Heisenberg Model

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1. INTRODUCTION

The success of equilibrium statistical mechanics in describing thermal properties of matter can be gauged through two of its robust predictions- the emergent phenomena and universality [1, 2]. However, recent experiments ranging from polariton condensates in context of semiconductor quantum wells in optical cavities [3–5], arrays of micro cavities [6] to trapped ions [7, 8], optomechanical setups [9, 10] and strongly interacting Rydberg polaritons [11, 12] to explore the bulk behavior of ultracold matter in presence of drives and dissipations lead to the breakdown of traditional equilibrium techniques. At microscopic scale, the very symmetry responsible for implementing the thermal order is broken down due to presence of drives and this breakdown is manifested in resulting breakdown of detailed balance principle [13–15]. In addition to coherent dynamics governed by Hamiltonian the above mentioned systems are also driven by dissipation requiring non-conventional evolution schemes characterised by competition between drives and dissipations. The many-body stationary states of novel evolutionary schemes emerge as new non-equilibrium phases of matter [16–20]. At macroscopic scale, in order to fix the notion of universality for emergent non- equilibrium phases, the fundamental challenge remains to look for alternatives for equilibrium notions like temperature, free energy and entropy which become vaguely defined for non-equilibrium phases of matter [21]. So the analogs of equilibrium notions like temperature etc. need to emerge self-consistently as a result of dynamics of the model [20, 22–30]. Therefore, despite non-equilibrium ingredients these systems equilibrate effectively and as a result of competition between drives and dissipations effective temperature is identified with suitable combination of dissipative parameters of underlying model via fluctuation-dissipation relations. However, despite effective equilibration these systems also defy traditional equilibrium signatures measured through corresponding response functions The driven open quantum systems can be

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well described by microscopic master equations [31, 32], but the traditional techniques of quantum optics cannot be used efficiently We employ Lindblad Master equation and map it to the Schwinger-Keldysh (SK) functional integral formalism [14, 18, 33] to study the dissipative dynamics in an interacting spin model with long range interactions described by an anisotropic Heisenberg model[34, 35] and coupled to a non-Markovian bath. This approach has found numerous applications to driven-dissipative systems such as lossy polariton condensates [18, 19, 36] and driven atomic ensembles interacting with a cavity mode [20].

In general, a system of qubits can be represented by some interacting spin-half particles. The interaction between these qubits can be nearest neighbor on a given lattice [35] or fully connected in a sense all spins interact with each other [37]. In this work, we consider a fully connected model described by anisotropic antiferromagnetic Heisenberg model (IRHM) where each spin is coupled to a same bosonic bath. We also consider Lindblad dynamics via a collective spin flipping. Lindblad dynamics is essentially Markovian in nature and our aim is to understand the influence of both Markovian and non-Markovian effects on the underlying dynamics of the system.

The long range interactions are used to study the fully connected in the context of light harvesting complexes [38, 39]. An example of such model is Fenna-Mathews-Oslo model [38–40] for excitons, where the hopping energy is uniform and the system bath coupling is not very weak but of intermediate range [37, 40, 41]. Furthermore, these interactions can be produced in cavity quantum electrodynamics experiments [31, 42, 43].

This paper is organized as follows. In section 2, we introduce the IRHM coupled with a bosonic bath. Using Holestien-Primakoff (HP) transformations, we bosonize IRHM model and map it to a self-interacting bosonic mode and find the parameter values where HP transformation breaks down. The complete Hamiltonian becomes Dicke model with non-linearities. In next section 3, we make use of SK functional integral formalism to study the steady state solutions of the equations of motion. We see that the critical coupling depends on the spectral density of the bath. In section 4, we study the dissipative spectrum beyond mean field level and analyze the effect of fluctuations on different observables. Finally, we conclude in section 5.

2. BOSONIZATION OF IRHM COUPLED WITH BOSONIC BATH

We consider a fully connected model of qubits represented by spin- $\frac{1}{2}$ particles interacting with each other through an infinite range Heisenberg antiferromagnetic exchange interaction H_{IRHM} [34, 37, 41] with anisotropy in the longitudinal channel, and coupled to a non-Markovian bath:

$$H = H_{IRHM} + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{\sqrt{N}}\sum_{i,k} S_i^x\left(g_k b_k + g_k^* b_k^\dagger\right) \tag{1}$$

Where

$$H_{IRHM} = \frac{J}{N}\sum_{i,j>i}[S_i^x S_j^x + S_i^y S_j^y + \Delta\ S_i^z S_j^z] \tag{2}$$

where $J > 0$ sets the energy scale of the model, and Δ represents an anisotropic parameter. $\overrightarrow{S} = \frac{h}{2}\vec{\sigma}, \vec{\sigma}$ are Pauli matrices and $S_\pm = S^x \pm i\ S^y$ are the ladder operators. We note that $\left[H_{\mathrm{IRHM}}, \sum_i S^z{}_i\right] = 0$ and $\left[H_{\mathrm{IRHM}}, S_{Total}^2\right] = 0$, so that the Eigen states of H_{IRHM} are described by the total S_T and S_z values. The ground state corresponds to singlet state $S_T^z = 0$ and $S_T = 0$, which has been shown to form a new class of highly entangled resonating valence bond states. This model has its own importance in describing zigzag graphene nanodisc [35, 44–48], Lipkin-Meshkov-Glick model [49] for certain parameter range. For quantum computation and information using quantum dots, the spin states are being prepared, manipulated, and measured using rapid control of Heisenberg exchange interaction [50–52].

Next we define total spin operator $\vec{S} = \sum_i \vec{S}_i$ and bosonize the H_{IRHM} using Holestien-Primakoff transformations [53]:

$$S^+ = \sqrt{N - a^\dagger a a} \tag{3}$$

$$S^- = a^\dagger \sqrt{N - a^\dagger a} \tag{4}$$

$$S^z = \frac{N}{2} - a^\dagger a \tag{5}$$

with N as the total number of spin-1/2 particles. Therefore, H_{IRHM} is mapped to a bosonic mode with non-linearities at various orders of $\frac{1}{N}$:

$$H_a = \frac{J}{2}\left(1 - \Delta - \frac{1}{N}\right)a^\dagger a + \frac{J}{2N}\left(\Delta - 1 + \frac{1}{2N}\right)\left(a^\dagger a\right)^2 + \frac{J}{4N^2}(a^\dagger a)^3 + \cdots \tag{6}$$

In the thermodynamic limit, we see that the $H_a \backsim \frac{J}{2}\ (1 - \Delta)a^\dagger a$, $\Delta = 1$ therefore, breaks the validity of the HP transformation as H_a vanishes in this limit, and at this point ground state becomes degenerate at lowest order of perturbation. Restricting to the case of $\Delta < 1$ and retaining the $O(\frac{1}{N})$ terms, we see that the coefficient of quartic term $\left(a^\dagger a\right)^2$ becomes negative for large N implying the instability of the bosonic mode. Therefore, for the stable ground state in the finite N limit, we require sextic term which is of the $O(\frac{1}{N^2})$ with positive coefficient (eqn. 6). Next, we see that for $J\Delta \gg 1$, even the coefficient of quartic term becomes positive, the system does not have a stable ground state in thermodynamic limit. This breakdown of HP transformation can mainly be attributed to different types of phase transitions occurring in the model as we vary Δ from $-\infty$ through 0 to $+\infty$; and to distance independent nature of the interactions in contrast to nearest neighbor XXZ model. Furthermore, we are interested in effect of bath in the thermodynamic limit of the model and it suffices to take $0 < \Delta < 1$. Therefore, the total Hamiltonian given by equation (1) as

$$H_{eff} = H_a + \sum_k \omega_k b_k^\dagger b_k + \frac{1}{2}\sum_k (a + a^\dagger)(g_k b_k + g_k^* b_k^\dagger) \quad (7)$$

This is just Dicke model [54] with non-linearities, and coupled to a multimode bath. The above model possess z_2--symmetry. In strong coupling regime within thermodynamic limit, the ground state of the above model breaks this z_2-symmetry and exhibits a phase transition to phase with $\langle a \rangle \neq 0$.

Next, we assume a dissipative process in addition to the coherent dynamics represented by the Hamiltonian in equation 7, due to spin flipping (spontaneous emission) at site i from $|\uparrow\rangle$ to $|\downarrow\rangle$ at a rate of κ, represented by Lindblad master equation:

$$\frac{d\rho_s}{dt} = -i[H_{IRHM}, \rho_s] + \frac{\kappa}{N}\sum_{i,j}[2 S_i^+ \rho_s S_j^- - \{S_i^+ S_j^-, \rho_s\}] \quad (8)$$

$$= -i [H_a, \rho_s] + \kappa [2a\rho_s a^\dagger - \{a^\dagger a, \rho_s\}] \quad (9)$$

where in second line we have used Holestien-Primakoff transformations, ρ_s is the density matrix corresponding to a -fields.

3. SCHWINGER-KELDYSH FIELD THEORY

In this section, we use SK field theoretic technique to study the dynamics in the model considered. The SK field theory is the path integral representation of the time evolved density matrix $Z = Tr\ \rho(t)$ on a closed time contour with fields defined along two branches called as forward (backward) time branch $(\pm)$, such that both branches meet at $t = \infty$. The partition function Z can be therefore written for some field $\phi(x)$ as (with $\bar{\phi}$ as the conjugate of ϕ) as:

$$Z = \int D[\bar{\phi}_+, \phi_+]D[\bar{\phi}_-, \phi_-]\ e^{iS_{SK}[\bar{\phi}_+, \phi_+, \bar{\phi}_-, \phi_-]} \quad (10)$$

Where Schwinger- Keldysh action for the total system plus bath including Lindblad dynamics is $S_{SK} = S_0 + S_D$ where S_0 is action for Hamiltonian dynamics:

$$S_0 = \sum_{\eta=\pm}\eta \int dx\ dt\ [\bar{\phi}_\eta i\partial_t \phi_\eta - H(\bar{\phi}_\eta, \phi_\eta)] \quad (11)$$

The action corresponding to dissipation due to Lindblad operator is given by S_D:

$$S_D = -i\kappa \int dx\ dt\ [2\phi_+\bar{\phi}_- - (\bar{\phi}_+\phi_+ + \bar{\phi}_-\ \phi_-)] \quad (12)$$

Therefore, for the model considerations, we write SK action as:

$$S_{SK} = S_a + S_b + S_{ab}$$

For a-type fields (S_a as action) we have

$$S_a = \int dt\ \left[\sum_{\sigma=\pm}\sigma[\ \bar{\phi}_\sigma(i\ \partial_t - \omega_0)\phi_\sigma + \frac{\lambda}{N}(\bar{\phi}_\sigma\phi_\sigma)^2 + \frac{J}{4N^2}(\bar{\phi}_\sigma\phi_\sigma)^3] - i\kappa(2\phi_+\bar{\phi}_- - \bar{\phi}_+\phi_+ - \bar{\phi}_-\ \phi_-\right] \quad (13)$$

Here $\omega_0 = \frac{J}{2}\left(1 - \Delta - \frac{1}{N}\right)$ and $\lambda = \frac{J}{2}(-1 + \Delta + \frac{1}{2N})$. ϕ represents the bosonic coherent state of a-type bosons, $\bar{\phi}$ represents the complex conjugate of ϕ. Plus (minus) signs refers to the field defined on forward (backward) branch of Keldysh contour. Similarly, if ψ represents the bosonic coherent state of b-type bosons (S_b as action), we can write as:

$$S_b = \int dt \sum_k \sum_{\sigma=\pm} \sigma[\bar{\psi}_{k\sigma}(i\partial_t - \omega_0)\psi_{k\sigma}$$

$$S_{ab} = -\frac{1}{2}\int dt \sum_k g_k \sum_{\sigma=\pm}\sigma\,(\bar{\phi}_\sigma + \phi_\sigma)(\bar{\psi}_{k\sigma} + \psi_{k\sigma}) \quad (14)$$

Next, we implement Keldysh rotation defined as: $\phi_{cl} = \frac{\phi_+ + \phi_-}{\sqrt{2}}$, $\phi_q = \frac{\phi_+ - \phi_-}{\sqrt{2}}$. The subscripts cl and q stand for the classical and the quantum components of the fields, respectively, because the first one can acquire expectation value while the second one cannot. In this basis, with the same transformations for ψ_k - field as well, we get:

$$S_a = \int dt\ \left[\begin{pmatrix}\bar{\phi}_{cl}(t) & \bar{\phi}_q(t)\end{pmatrix}\begin{pmatrix}0 & i\partial_t - \omega_0 - i\kappa \\ i\partial_t - \omega_0 + i\kappa & 2i\kappa\end{pmatrix}\begin{pmatrix}\phi_{cl}(t) \\ \phi_q(t)\end{pmatrix}\right.$$

$$+ \frac{\lambda}{2N}\left(|\phi_{cl}|^2 + |\phi_q|^2\right)(\ \bar{\phi}_{cl}\ \phi_q + \phi_{cl}\bar{\phi}_q)$$

$$+\frac{J}{4\sqrt{2}N^2}[(\bar{\phi}_{cl}\ \phi_q)^3 + (\bar{\phi}_q\phi_{cl})^3$$

$$+3\left(|\phi_{cl}|^4 + |\phi_q|^4 + 3|\phi_{cl}|^2|\phi_q|^2\right)(\bar{\phi}_{cl}\ \phi_q + \phi_{cl}\bar{\phi}_q) \quad (15)$$

$$S_b = \sum_k \int dt \left(\bar{\psi}_{kcl}(t)\bar{\psi}_{kq}(t)\right)\begin{pmatrix}0 & i\partial_t - \omega_k - i\epsilon \\ i\partial_t - \omega_k + i\epsilon & 2i\epsilon\end{pmatrix}\begin{pmatrix}\psi_{kcl}(t) \\ \psi_{kq}(t)\end{pmatrix} \quad (16)$$

$$S_{ab} = -\frac{1}{2}\sum_k g_k \int dt\ [\ (\bar{\phi}_{cl} + \phi_{cl})\ (\ \bar{\psi}_{kq} + \psi_{kq}) + (\bar{\psi}_{kcl} + \psi_{kcl})\ (\bar{\phi}_q + \phi_q)\ ] \quad (17)$$

where ϵ is the regularization parameter. Markovian dissipation is defined by the frequency independent part of Keldysh component [20]. Next we perform saddle point approximation by varying action (with respect to quantum component of the fields, i.e. $\frac{\delta S}{\delta \phi_q} = 0$ and $\frac{\delta S}{\delta \psi_{kq}} = 0$ at $\phi_{cl} = \phi_0, \phi_q = 0$ and $\psi_{kcl} = \psi_{k0}$, $\psi_{kq} = 0$ and get:

$$(-\omega_0 + i\kappa)\phi_0 + \frac{\lambda}{2N}|\phi_0|^2\phi_0 + \frac{3J}{4\sqrt{2}N^2}|\phi_0|^4\phi_0 - \frac{1}{2}\sum_k g_k(\bar{\psi}_{k0} + \psi_{k0}) = 0$$

$$(-\omega_0 + i\epsilon)\psi_{k0} - \frac{1}{2}g_k(\bar{\phi}_0 + \phi_0) = 0 \quad (18)$$

In order to solve above equations, we define bath spectral density $J(\omega) = \sum_k g_k^2 \delta(\omega - \omega_k)$.

We consider the following general form of $J(\omega)$ with Drude-Lorentz cutoff:

$$J(\omega) = 2\pi\gamma\omega\left(\frac{\omega}{\Omega}\right)^{s-1}\frac{\Omega}{\omega^2+\Omega^2} \quad (19)$$

With γ as the effective coupling between system and bath, Ω is the cutoff frequency. $s = 1$ correspond to Ohmic bath $0 < s < 1$ and $s > 1$ are called sub-ohmic and super-ohmic baths respectively. However, we will work with ohmic bath $s = 1$ for simplicity. Using this form of spectral density, we see that the saddle point equations 18 at $O\left(\frac{1}{N}\right)$ admit a trivial solution $\phi_0 = 0$ for symmetric state and a non-trivial solution $\phi_0 \neq 0$ for symmetry broken state which is given by

$$|\phi_0| = \pm\sqrt{\frac{N\pi}{\lambda}}\ (\gamma_0 - \gamma)^{\frac{1}{2}} \quad (20)$$

Where $\gamma_0 = \frac{1}{\pi}\frac{\omega_0^2+\kappa^2}{\omega_0}$ is the critical coupling.

Now we evaluate the various correlation function corresponding to ϕ-field within the mean field level. In the thermodynamic limit $N \to \infty$, the contribution from $O\left(\frac{1}{N}\right)$ terms can be ignored. We first eliminate the ψ-field using Gaussian integration. Defining $\Phi_{cl/q} = \begin{pmatrix}\phi_{cl/q}(\omega)\\ \bar{\phi}_{cl/q}(-\omega)\end{pmatrix}$ and $\Psi_{cl/q} = \begin{pmatrix}\psi_{cl/q}(\omega)\\ \bar{\psi}_{cl/q}(-\omega)\end{pmatrix}$ such that Keldysh-Nambu spinor is defined as $\eta_8(\omega) = [\Phi_{cl}\Psi_{kcl}\Phi_q\Psi_{kq}]^T$. Using the notation $\int_\omega = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}$ and $\phi_{cl/q}(t) = \int_\omega e^{-i\omega t}\phi_{cl/q}(\omega)$ as the Fourier transform of the ϕ-field, we integrate out ψ-field to get the following effective action for ϕ-field:

$$S_{eff} = \int_\omega \eta_4^\dagger(\omega)\begin{pmatrix}0 & [G_{2\times2}^A]^{-1}(\omega)\\ [G_{2\times2}^R]^{-1}(\omega) & D_{2\times2}^K\end{pmatrix}\eta_4(\omega) \quad (21)$$

Where $\eta_4(\omega) = \begin{pmatrix}\Phi_{cl}(\omega)\\ \Phi_q(\omega)\end{pmatrix}$, $D_{2\times2}^K = diag\ (2i\kappa, 2i\kappa)$. The retarded Green's function is given by:

$$[G_{2\times2}^R]^{-1}(\omega) = \begin{pmatrix}\omega - \omega_0 + i\kappa + \sum^R(\omega) & \sum^R(\omega)\\ [\sum^R(-\omega)]^* & -\omega - \omega_0 - i\kappa + [\sum^R(-\omega)]^*\end{pmatrix} \quad (22)$$

Here $\sum^R(\omega) = [\sum^R(-\omega)]^* = -\frac{1}{2}\sum_k\frac{|g_k|^2\omega_k}{\omega^2-\omega_k^2}$ is the self-energy function. Thus it is evident that self-energy depends on the density of bath states. Using the density of states given by equation 19, we write the self-energy function $\sum(\omega) \equiv \sum^R(\omega)$ for Ohmic case as:

$$\sum(\omega) = \frac{\pi}{2}\gamma\ \frac{\Omega^2}{\omega^2+\Omega^2} \quad (23)$$

The characteristic frequencies of the system are defined by the zeros of the determinant $[G_{2\times2}^R]^{-1}(\omega)$ those correspond to the poles of the response function $G_{2\times2}^R(\omega)$. Since Green's function possess the symmetry that $\sigma_x G_{2\times2}^R(\omega)\sigma_x = [G_{2\times2}^R(-\omega)]^*$, so that the roots come into pairs with opposite real parts or are purely imaginary. Thus the dispersion of dissipative modes are given by $det[G_{2\times2}^R]^{-1}(\omega) = 0$ which implies:

$$\omega = -ik \pm\sqrt{{\omega_0}^2 - 2\omega_0\sum(\omega)} \quad (24)$$

Figure 1 is the plot of real and imaginary parts of the roots of the above characteristic equation for different values of k with anisotropic parameter $\Delta = 0.7$, $J = 1$. We see that for no spin-flipping case $k = 0$, we have all the roots vanishing at transition point $\frac{\gamma}{\gamma_0} = 1$ as expected. As we increase value of k, different modes hybridize and get shifted in the opposite directions . On approach to transition point two solutions become purely imaginary and correspond to damped modes as shown by blue and black curves in the Figure 1(b), and (c) . While at transition point only one mode shown by red curve in Figure 1 (b), and (c) vanish and thus making the system dynamically unstable.

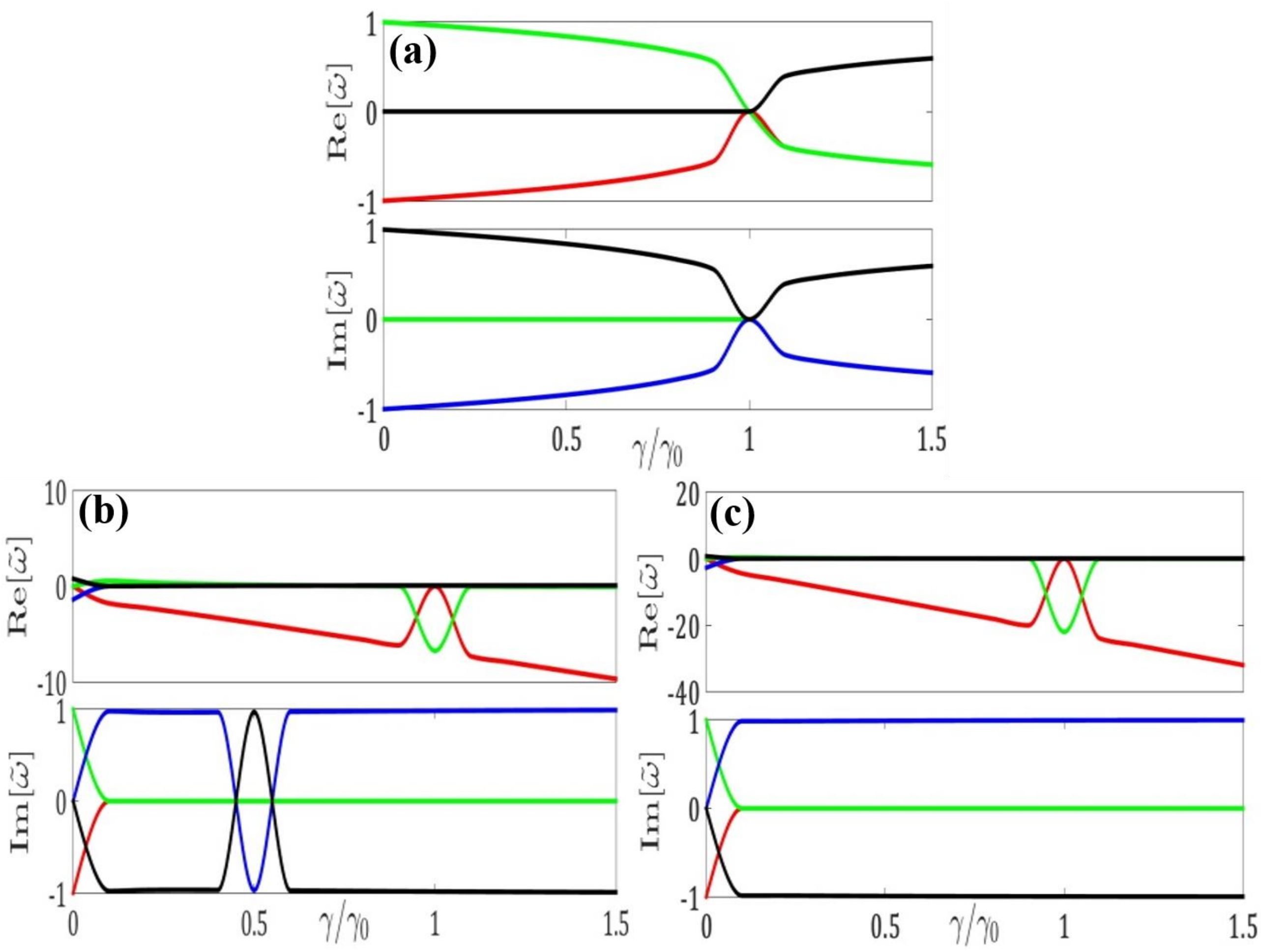

Fig. 1. Real and imaginary part of the roots of the equation 3.17 pertaining to characteristic frequencies of the system $\tilde{\omega} = \frac{\omega}{\Omega}$. The chosen values are (a) $k = 0$, (b) $k = 0.3$, and (c) $k = 1$. We see that one of the Eigen modes (red curve) vanish at $\gamma = \gamma_0$ for some finite k value.

3.1. Correlation Functions

The physically measurable quantities are correlation functions. The spectral response function $A(\omega)$ encodes the systems response to the active, external perturbations. It is defined as:

$$A(\omega) = i[G^R(\omega) - G^A(\omega)] \tag{25}$$

In the present case, we write $A(\omega) = -2\text{Im}G^R(\omega)$ and is given by

$$A(\omega) = \frac{2[(\omega^2+\kappa^2+\omega_0^2+2\omega\omega_0)\kappa-2\kappa(\omega_0+\omega)\Sigma]}{(\omega^2-\kappa^2-\omega_0^2+2\omega_0\ \Sigma)+4\omega^2\kappa^2} \tag{26}$$

At $\gamma = 0$, we see from the Figure 2 that $A(\omega)$ has Lorentzian shape centered at ω_0. As γ increases towards γ_0, the Lorentzian peak gets shifted towards low frequency mode $\omega = 0$ at transition point.

The correlation function encodes the systems internal correlations and is defined as:

$$C(t,t') = \langle\{\hat{a}(t), \hat{a}^\dagger(t')\}\rangle = iG^K(t,t') \tag{27}$$

In steady state, we write

$$C = 2\langle a^\dagger a\rangle + 1 = i\ \int \frac{d\omega}{2\pi} G^K(\omega) \tag{28}$$

With

$$iG^K(\omega) = \frac{2\kappa[(\omega+\omega_0-\Sigma)^2+\kappa^2+\ \Sigma^2]}{(\omega^2-\kappa^2-\omega_0^2+2\omega_0\ \Sigma)^2\ \ +4\omega^2\kappa^2} \tag{29}$$

For a decaying bosonic mode with no coupling to the bath i.e. $\gamma = 0$, we see from the equations 26 and 28 that $C(\omega) = A(\omega)$, and the steady state boson density $\langle a^\dagger a\rangle = 0$, which corresponds to the vacuum of the ϕ-field. We see from the

figure 2 that there occurs divergence $C(\tilde{\omega})$ for $\tilde{\omega} = 0$ at transition point $\frac{\gamma}{\gamma_0} = 1$ resulting in the divergence of occupation density of bosons, see for example Figure 3.
The average number of bosons diverge at transition point as:

$$2\langle a^\dagger a\rangle + 1 \sim |\gamma_0 - \gamma|^{-\alpha} \tag{30}$$

With $\alpha = 0, 1$ for $\kappa = 0$ and $\kappa \neq 0$ respectively.

3.2. Effective Temperature

The response and correlation functions allows us to define a fluctuation-dissipation relationship by introducing distribution function $F(\omega)$:

$$G^K(\omega) = G^R(\omega)F(\omega) - F(\omega)G^A(\omega) \tag{31}$$

The distribution function has the form $F_{eq}(\omega) = 2n(\omega) + 1 = \coth(\frac{\omega}{2T})$ with $n(\omega) = \frac{1}{e^{\beta\omega} - 1}$ in thermal equilibrium. In non-equilibrium setting here, the notion of effective temperature is determined through the low frequency analysis of Eigen values of the distribution function $F(\omega)$. For our problem, we write:

$$F(\omega) = \sigma^2 - \frac{1}{2\omega}\sum_k \frac{g_k^2\omega_k}{\omega^2 - \omega_k^2}\sigma^x \tag{32}$$

Where σ^z and σ^x are Pauli spin matrices. The Eigen values of $F(\omega)$ are given by:

$$\lambda_\pm(\omega) = \pm\sqrt{1 + \left(\frac{\Sigma(\omega)}{\omega}\right)^2} \tag{33}$$

Therefore, in the long wavelength we write $F(\omega) \sim \frac{2T}{\omega}$. We see from equation 33, in this limit, Eigen values $\lambda_\pm$ diverges as $\frac{1}{\omega}$. The effective temperature T_{eff} is given by the dimensional coefficient of $\frac{1}{\omega}$ in the long wavelength limit. Therefore, we see that $T_{eff} = \gamma$ and is independent of the decay rate κ, cutoff frequency Ω of the bath. It can be shown true for all cases of spectral densities wit Drude-Lorentz cutoff. Moreover, if we chose exponential cutoff for the bath spectral density, we can show that effective temperature depends on cutoff frequency as well besides coupling γ. This effective temperature in comparison to equilibrium is not an external parameter but an intrinsic quantity that arises due to interplay of unitary and dissipative dynamics.

4. FLUCTUATIONS OVER MEAN FIELD

Having found out the mean field solution, we now consider the stability of these solutions to small fluctuations around mean field. We therefore add small fluctuations at tree level by taking: $\phi_{cl} \to \phi_0 + \delta\phi$ and $\phi_q \to \delta\phi_q$. Therefore, from equation 21 and taking $O(\frac{1}{N})$ terms into account, we write:

$$\tilde{S} = \int_\omega \delta\eta_4^\dagger(\omega) \begin{pmatrix} 0 & [\tilde{G}^A_{2\times2}]^{-1}(\omega) \\ [\tilde{G}^R_{2\times2}]^{-1}(\omega) & D^K \end{pmatrix} \delta\eta_4(\omega)$$

$$-\frac{\lambda}{2N}\int_t \left[(2\phi_0|\phi_{cl}|^2 + \phi_0^*\phi_{cl}^2)\phi_q^* + \left(|\phi_{cl}|^2|\phi_q|^2\right)\phi_{cl}\phi_q^* + c.c\right] \tag{34}$$

With $\delta\eta_4(\omega) = \begin{pmatrix} \delta\phi_{cl}(\omega) \\ \delta\phi_q(\omega) \end{pmatrix}$ and

$$[\tilde{G}^R_{2\times2}]^{-1}(\omega) = \begin{pmatrix} \omega - \omega_0 + i\kappa + \Sigma(\omega) - \frac{\lambda}{N}|\phi_0|^2 & \Sigma(\omega) - \frac{\lambda}{2N}\phi_0^2 \\ \Sigma(\omega) - \frac{\lambda}{2N}\phi_0^{*2} & -\omega - \omega_0 - i\kappa + \Sigma(\omega) - \frac{\lambda}{N}|\phi_0|^2 \end{pmatrix} \tag{35}$$

while contribution to action at $O\left(\frac{1}{N}\right)$ are due to cubic and quartic terms. Thus we observe that the fluctuations vanish in the thermodynamic limit $N \to \infty$. The poles of the retarded Green's function, give the spectrum of excitations, while the signs of their imaginary parts determine whether the proposed mean-field steady state is stable. A positive imaginary part of the spectrum implies the instability to mean field solution. Thus, to find the dissipative spectrum of fluctuations, we solve $\det[\tilde{G}^R_{2\times2}](\omega) = 0$ and get:

$$\omega = -i\kappa \pm \sqrt{(\omega_0^2 - 2\omega_0\Sigma) - \frac{\lambda}{2N}[(\phi_0 - \phi_0^*)^2\Sigma + 2\omega|\phi_0|^2]} \tag{36}$$

Therefore, in the limit of $N \to \infty$, the fluctuations are washed away, and we retain the same mean field spectrum given in equation 24. Next, we analyze the effect of fluctuations on the distribution matrix $F(\omega)$ that provides the information regarding effective temperature. From fluctuation-dissipation relation 31, we can write

$$F(\omega) = \sigma^Z + \frac{1}{\omega}\left[\Sigma(\omega) - \frac{\lambda}{4N}(\phi_0^2 + \phi_0^{*2})\right]\sigma^x \tag{37}$$

which has the same form in thermodynamic limit $N \to \infty$ as defined in equation 32. Thus fluctuations due to finite number of particles N reduce the effective temperature.
Now, we take into account the contribution of cubic and quartic terms in the effective action. In principle we can sum up to all orders of perturbation and get the following equation

$$[G_0^{-1} - \Sigma] \circ \mathcal{G} = I_{2\times2} \tag{38}$$

Where G_0^{-1} is the bare Greens function, $\mathcal{G}$ is the dressed Greens function due to the interactions and the self-energy matrix is $\Sigma = \begin{pmatrix} 0 & \Sigma^A \\ \Sigma^R & \Sigma^K \end{pmatrix}$. However, we restrict here to the qualitative ideas, whereas the full details of effects of interactions are treated separately [55] within the renormalization group approach in Keldysh space.

We consider the effect of fluctuations at first order of $\frac{\lambda}{N}$. The cubic terms at this order are:

$$\int_t [2\phi_0\phi_{cl}^2\phi_q^* + \phi_0^*\phi_{cl}^2\phi_q + c.c.].$$

This term breaks the $\mathcal{Z}_2$ – symmetry, $\phi_{cl/q} \rightarrow -\phi_{cl/q}$ and can be treated as the external "magnetic" field term. In general, the fluctuations can modify the position of the critical point and these terms serve the corrections to the mean field position of the phase transition. However, we can eliminate these odd order terms by applying the external drive. This kind of situation also arises in the liquid-gas transition, where there is no obvious symmetry, however, one can choose parameters such as density to eliminate odd terms. This phase transition, despite the absence of symmetry, is of the Ising type [55]. A similar conclusion holds if we take fluctuations at higher order of λ/N . Moreover, we can show [55] that this model undergoes a second order thermodynamic phase transition of ϕ^4- theory with $\mathcal{Z}_2$-symmetry. We thus conclude that the driven-dissipative model considered here undergoes a continuous Ising- type phase transition.

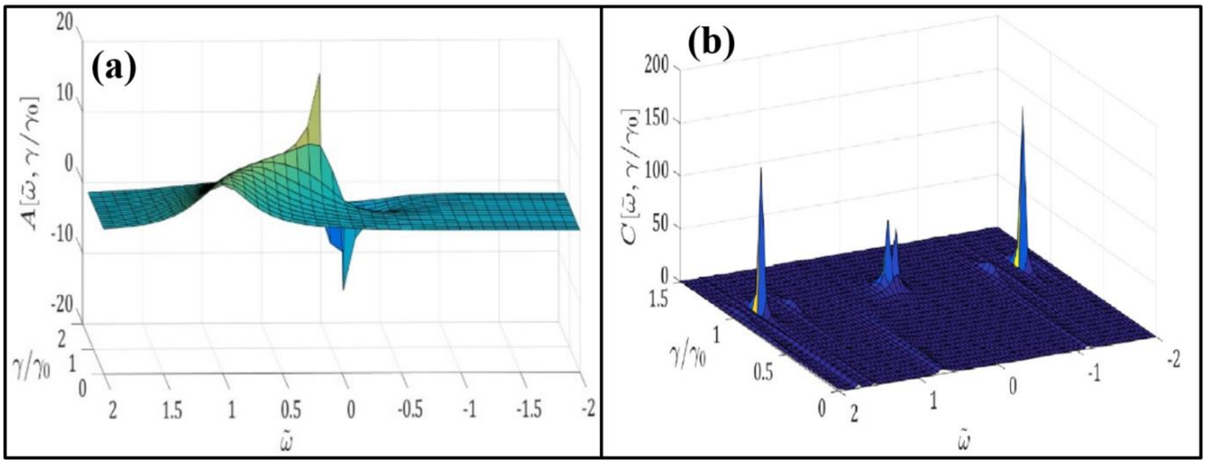

Fig. 2. (a) Spectral response function $A(\tilde{\omega}, \frac{\gamma}{\gamma_0})$ as a function of $\frac{\gamma}{\gamma_0}$and $\tilde{\omega} = \frac{\omega}{\Omega}$ for= $0.3, \omega_0 = 1$. The Lorentzian peak at $\gamma = 0$ is shifted towards low frequency mode at transition point. (b) Correlation Function $C(\tilde{\omega}, \frac{\gamma}{\gamma_0})$ as a function of $\frac{\gamma}{\gamma_0}$ and $\tilde{\omega} = \frac{\omega}{\Omega}$ for $k = 0.3, \omega_0 = 0$.

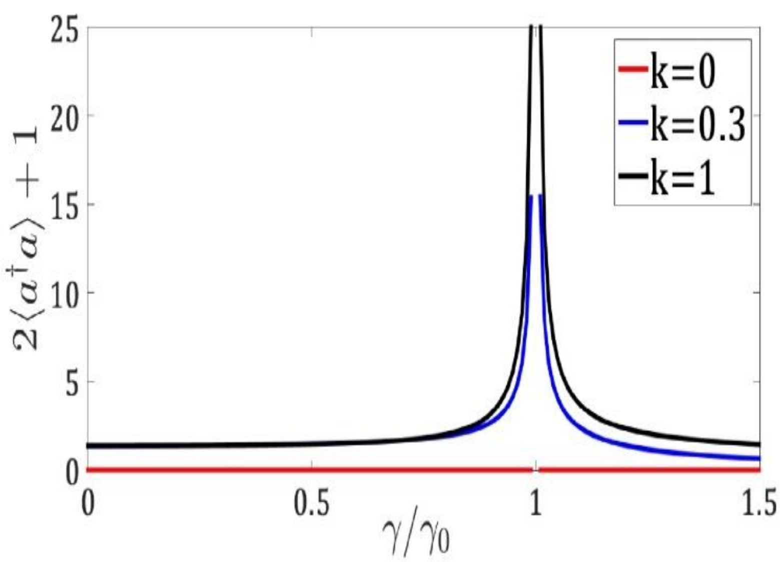

Fig. 3. Steady state number density for different values of κ, and $\omega_0 = 1$. The distribution function diverges as $|\gamma - \gamma_0|^{-\alpha}$ with $\alpha = 0$ for $\kappa = 0$ and $\alpha = 0$ for $\neq 0$.

5. CONCLUSION

In conclusion, we have analyzed the non-equilibrium dynamics in a long-range interaction Heisenberg model coupled to a bath and driven by dissipation at each site due to spin flipping (spontaneous emission). Our findings indicate that the Holstein-Primakoff transformation cannot be faithfully applied across the entire parameter range of the model. Within a limited domain of parameter values, we have successfully mapped the IRHM to a multimode Dicke model with non-linearities. Utilizing the Keldysh field theory, we demonstrated that in the thermodynamic limit, the boson density of the system exhibits power law behavior, with the critical exponent depending on the values of the decay constant κ and the type of spectral density employed. Moreover, we have shown that an effective temperature arises due to dissipation and depends linearly on the effective coupling γ, independent of the bath's cutoff frequency across a wide class of bath spectral densities. Fluctuations due to cubic field terms in the perturbation expansion violate Z_2-symmetry and modify the mean field critical point. However, near the steady state, the dynamics are generally described by a thermodynamic universality class of ϕ^4 theory, as proposed by Landau and Ginzburg. The emergent thermal character of driven-dissipative systems can be expected as quantum coherence is lost to dissipation.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interests.

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